TRINITY COLLEGE

YEAR 12
MATHEMATICS
METHODS

Test 2, 2023
Section One: Calculator Free
Applications of Anti-derivative, FTOC & DRVs

STUDENT'S NAME: _____

DATE: Monday 8th May **TIME:** 25 minutes **MARKS**: 28
ASSESSMENT %: 10

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items:

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

Question 1 **(6 marks)**

Determine

(a) $\int (8x-6)e^{2x^2-3x+1}dx$ (2 marks)

(b) $\int\left(\sin\frac{3x}{2}-2x\cos\frac{2\pi}{5}\right)dx$ (2 marks)

(c) $\frac{d}{dx}\int_x^1 \frac{2}{3t^3-1}dt$ (2 marks)

Question 2 **(6 marks)**

Determine, with reasoning, whether each of the following represent a discrete random variable.

(a)

x	0	1.5	2	3	4
$P(X=x)$	0.3	0.1	0.4	0.05	0.15

(2 marks)

(b)

x	-2	-1	1	3	5
$P(X \leq x)$	0	0	0.2	0.6	1

(2 marks)

(c) $P(X = x) = \left(\frac{1}{2}\right)^x$; $x = 1, 2, 3, 4....$ (2 marks)

Question 3 **(5 marks)**

(a) Determine $\frac{d}{dx}\left(2x\sin(3x)\right)$ (2 marks)

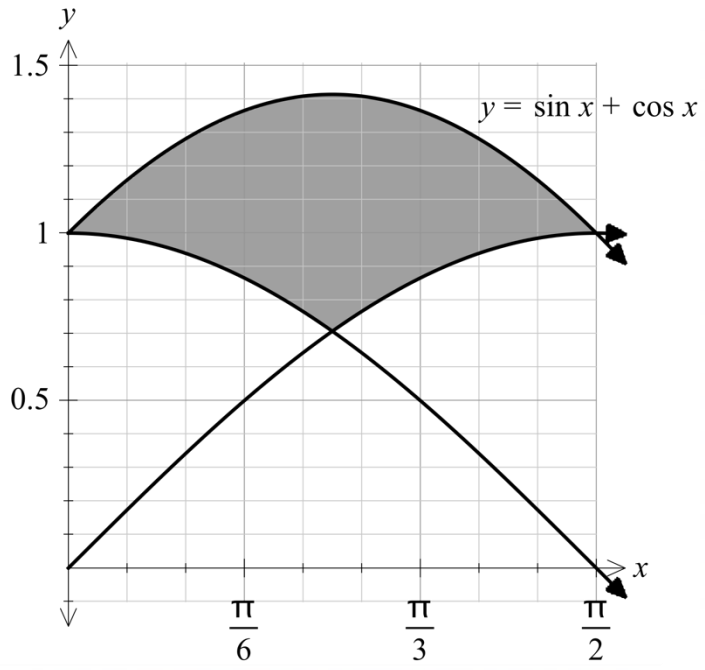
(b) Using your answer from (a) or otherwise, determine $\int 6x\cos(3x)dx$ (3 marks)

Question 4 **(4 marks)**

The graph on the right shows three functions:

$f(x) = \sin x$
$g(x) = \cos x$
$h(x) = \sin x + \cos x$

Determine the exact value of the shaded area.



Question 5 **(7 marks)**

(a) Each of the following represent discrete probability functions. Determine the value of *k* for each.

(i) $P(x)=\frac{1}{k};\ x=1,2,3,...12$ (1 mark)

(ii)

x	1	2	3	5	7
$P(X=x)$	$2k$	k	k	$5k$	$6k$

(2 marks)

(b) The random variable *X* has probability distribution function $p(x)$ defined by $p(x)=\frac{x+2}{k}$ for $x=-1,0,1$ and 2.

(i) Determine the value of k. (2 marks)

(ii) Calculate $P(X=0|X\neq 1)$. (2 marks)

END OF QUESTIONS

TRINITY COLLEGE

YEAR 12
MATHEMATICS
METHODS

Test 2, 2023
Section Two: Calculator Allowed
Applications of Anti-derivative, FTOC & DRVs

STUDENT'S NAME: _____

DATE: Monday 8th March **TIME:** 25 minutes **MARKS**: 31
ASSESSMENT %: 10

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items: 1 A4 page notes, Classpad, Scientific Calculator

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

Question 6 **(4 marks)**

Harry fires an arrow at a target n times. The probability, p, of Harry hitting the target is constant and all shots are independent.

Let X be the number of times Harry hits the target in the n attempts.

The mean of X is 32 and the standard deviation is 4.

(a) State the distribution of X. (1 mark)

(b) Determine n and p. (3 marks)

Question 7 **(4 marks)**

The area bound by the curve $y = e^{x+1} + 2e$ and the x - axis between the values $x = 0$ and $x = k$ is equal to $e^3 - e^2$ square units.

Determine the value of k given $k > 1$.

Question 8 **(7 marks)**

At time $t = 0$, a small body P is at the origin O and is moving with a velocity of 18 ms^{-1}. The acceleration of P for $t \geq 0$ is given by $a = \frac{-3}{\sqrt{t+4}}\ ms^{-2}$.

(a) Determine the velocity of P when $t = 5$. (4 marks)

(b) Determine the distance of P from O at the instant P is stationary. (3 marks)

Question 8 **(9 marks)**

Studies in Britain have recorded that 1 in 100 eight-year-old children need at least one tooth removed caused by sugary drinks and severe tooth decay.
A typical primary school class of 24 eight-year-olds are investigated for the need to remove at least one tooth.

Determine the probability of:

(a) 2 students needing at least one tooth removed. (1 mark)

(b) No students requiring the removal of any teeth. (1 mark)

(c) At least one student requiring the removal of at least one tooth. (2 marks)

(d) Less than 4 students requiring the removal of at least one tooth given that at least one student required tooth removal. (2 marks)

Of the thirteen-year-olds in Britain requiring tooth removal, the probability of them requiring just one tooth out of their 32 permanent teeth removed is 5%.

(e) Calculate the probability of a permanent tooth in a thirteen-year-old needing removal. (3 marks)

Question 9 **(7 marks)**

The discrete random variable X has the following probability distribution:

x	1	2	3	4	5
$P(X = x)$	0.1	a	0.3	0.25	b

(a) Determine the values of a and b if the expected value, $E(X) = 3.3$ (3 marks)

(b) Determine the variance, $Var(x)$. (2 marks)

(c) State the value of $E(X + 5)$ (1 mark)

(d) State the value of $Var(5 - 2X)$ (1 mark)

END OF QUESTIONS